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THEORY OF A FLAT SUBMERGED JET OF A NON-NEWTONIAN LIQUID WITH A POWER RHEOLOGICAL LAW

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The character of the propagation of shear perturbations in non-Newtonian liquids with a power rheological law [1]

$$\sigma_{ij} = 2k\,(f_{\alpha\beta}f_{\alpha\beta})^{(n-1)/2}f_{ij} \tag{1}$$

is essentially determined by the value of the exponent n in (1), where σ_{ij} is the deviator of the stress tensor; f_{ij} is the tensor of the deformation rates; k and n are rheological constants of the medium. With the terminology adopted, media with $n>1$ are called dilatant, and with $n<1$ they are called pseudoplastic; the case $n=1$ corresponds to a Newtonian viscous liquid. It is well known that, in dilatant liquids, shear perturbations are propagated with a finite rate, whereas, in pseudoplastic and Newtonian viscous liquids, the rate of propagation of perturbations is infinite [2, 3]. As a result of this, there is a finite thickness of the boundary layer with laminar flow of a dilatant liquid past a flat semiinfinite plate. Actually, the finite thickness of the boundary layer in this case is explained by the fact that the shear perturbations, propagating with a finite velocity, are carried along the flow and emerge to the surface, at which the layer is formed only at a finite distance in the direction of its transverse coordinate. The inexact picture given in [4], unjustifiably excluded the fact of the finite thickness of the boundary layer in the case of "densifying" dilatant liquids with $1<n<2$. At the same time, the finite thickness of the boundary layer can be rigorously shown in the case of any given dilatant liquid with arbitrary values of $n>1$.

If, in dilatant liquids, the rate of propagation of shear perturbations is finite, a flat laminar jet immersed in such liquids should have a finite thickness, i.e., at a finite distance from the axis of the jet in the liquid there is a surface $y=y_\Phi(x)$ outside of which the longitudinal component of the velocity is equal to zero (see Fig. 1). This is connected with the fact that the jet brings into motion the liquid into which it flows out; in addition to the longitudinal, the liquid has a transverse component of the velocity, directed toward the axis of the jet. On the other hand, the rate of propagation of shear perturbations in dilatant liquids, connected with a change in the longitudinal component of the velocity, decreases with an increase in the distance from the source

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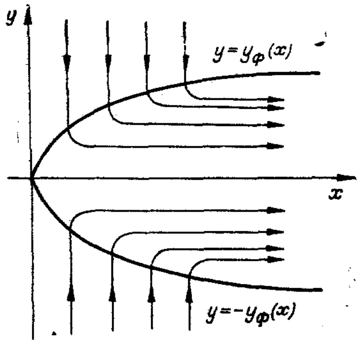

Fig. 1

of the perturbations, located in a submerged jet at its axis. It is therefore to be expected that, at a determined distance from the axis of the jet, the rate of propagation of the shear perturbations becomes equal to the transverse component of the velocity of the liquid, as a result of which the shear perturbations are propagated only to a finite distance from the axis of the jet. In other words, the front of the shear wave must stop in the liquid, and the shear perturbations penetrate into the liquid only to a finite distance from the axis of the jet.

The solution of the problem of a flat submerged jet of a power liquid was considered in the case of pseudoplastic liquids in [5-7]. However, no analytical solution was obtained for the case of dilatant liquids. It is natural therefore that, in this work, the fact of the spatial localization of the shear perturbations was not observed. In view of this the results of the present work supplement the considerations given in [5-7].

From an infinitely thin slit in the half-space $x > 0$ (see Fig. 1) filled with a power liquid (1) let there issue a liquid with a constant value of the momentum

$$\int_{-\infty}^{+\infty} \rho u^2 dy = 2I_0, \tag{2}$$

where ρ is the density of the liquid; $u(x, y)$ is the longitudinal component of the velocity. The flow of liquid is symmetrical with respect to the x axis and, for its description, it is sufficient to limit ourselves to the region $x, y \geq 0$. Then, for determination of the stream function

$$\psi(x, y) = a^{1/2(2-n)} x^{1/3n} \; f(\eta), \tag{3}$$

$$\eta = \left(\frac{1}{3na^{1/2}}\right)^{1/(2n-1)} \frac{y}{x^{2/3n}}, \quad a \equiv \frac{k}{\rho}, \quad x, y \geqslant 0$$

in the approximation of the boundary layer the following problem arises [8]:

$$(-1)^{n-1} \frac{d}{d\eta}\left(\frac{d^2 f}{d\eta^2}\right)^n + \frac{d}{d\eta}\left(f \frac{df}{d\eta}\right) = 0; \tag{4}$$

$$f(0) = \frac{d^2 f}{d\eta^2}(0) = \frac{df}{d\eta}(\infty) = 0, \tag{5}$$

as a result of whose solution, taking account of (3), both projections of the velocity $u(x, y)$ and $v(x, y)$ can be determined.

Integrating Eq. (4) twice taking account of the first two boundary bonditions (5) we have

$$\frac{df}{d\eta} = \left[\frac{2n-1}{n+1}\left(A^{(n+1)/n} - f^{(n+1)/n}\right)\right]^{n/(2n-1)}, \tag{6}$$

where A is an integration constant, whose value is determined below. Assuming

$$f(\eta) = A\xi^{n/(n+1)}, \tag{7}$$

after integration of (6) we can obtain the expression

$$\int_0^{\xi} \xi^{-1/(n+1)} (1-\xi)^{-n/(2n-1)} d\xi = \frac{[A^{2-n}(2n-1)^n (n+1)^{n-1}]^{1/(2n-1)}}{n} \eta, \tag{8}$$

using which the solution of problem (4), (5) is written in quadratures.

It is obvious that, with $\xi = 1$, $df/d\eta = 0$, the expression under the integral sign in (8) has a singularity, integrable if $n > 1$, and nonintegrable if $n \leq 1$. It therefore follows from (8) that, in the case of pseudoplastic and Newtonian viscous liquids, the third condition of (5) is satisfied with $\eta \to \infty$. Thus, expression (8) must be regarded as the solution of problem (4), (5) in the case $n \leq 1$ with the as yet undetermined constant A.

In the case of dilatant liquids, the third condition of (5) is found to be satisfied with $\eta = \eta_\Phi < \infty$:

$$\eta_\Phi = \frac{n}{[(2n-1)^n (n+1)^{n-1}]^{1/(2n-1)}} A^{(2-n)/(2n-1)} B\left[\frac{n}{n+1}, \frac{n-1}{2n-1}\right] \tag{9}$$

(B[p, q] is a beta function [9]).

Equation (6), which must be integrated during the process of the construction of the solution of the problem (4), (5), in addition to the partial solution $f_1(\eta)$, determined by expressions (7), has a special solution $f_2(\eta) = A$ [10]. Noting that the constant solution f = const satisfies Eq. (4) and the third boundary condition of (5), we construct the generalized solution of the problem (4), (5)

$$f(\eta) = \begin{cases} f_1(\eta) & \text{with } 0 \leqslant \eta \leqslant \eta_\Phi, \\ f_2(\eta) & \text{with } \eta_\Phi \leqslant \eta < \infty, \end{cases} \tag{10}$$

joined with $\eta = \eta_\Phi$ with a weak discontinuity from the partial and special solutions of Eq. (6).

Thus, a solution of problem (4), (5) has been constructed with $n > 1$, having a different analytical description with different values of η. The physical integration of solution (10) means that the longitudinal component of the velocity u(x, y) in the case of submerged jets of dilatant liquids varies completely inside of the spatially localized region $-y_\Phi(x) \leq y \leq y_\Phi(x)$ (see Fig. 1), outside of which it is equal to zero:

$$u(x, y) = \frac{\partial\psi}{\partial y} = 0, \quad v(x, y) = -\frac{\partial\psi}{\partial x} = -\frac{A}{3n} a^{1/2(2-n)} x^{(1-3n)/3n}$$

$$(0 < x < \infty, \ y_\Phi(x) < y < \infty).$$

The limits of the region of spatial localization of the shear perturbations are determined from (3), (9)

$$y_\Phi(x) = (3na^{1/2})^{1/(2n-1)} \ \eta_\Phi x^{2/3n}. \tag{11}$$

The integration constant A is calculated from the condition (2) taking account of (3), (6), (7); as a result, for all values of n we have

$$A = \left\{\frac{1}{2n-1}\left[\left(\frac{I_0}{\rho B\left[\frac{n}{n+1}, \frac{3n-1}{2n-1}\right]}\right)^{2n-1} \frac{3(n+1)^{3n-1}}{n^{2(n-1)} a^{(5n-4)/2(2-n)}}\right]^{1/n}\right\}^{1/3n}. \tag{12}$$

Since $B[p, q \to +0)] \to \infty$, then, from (9) and (11) it follows that, with $n \to 1+0$ $y_\Phi(x) \to \infty$, i.e., with a transition to the case of a Newtonian viscous liquid, the longitudinal component of the velocity u(x, y) varies, not in a localized region, but in the whole half-space $x > 0$.

There is another obvious limiting transition in expressions (9), (11), (12) to the case of a "limiting dilatant" liquid $n \to \infty$ [8]. In this case, $y_\Phi(x) \to 0$, and the jet issuing from the slit moves through a medium at rest, like a solid rod through an ideal nonviscous liquid.

In conclusion we note that the spatial localization of the shear perturbations in dilatant liquids is similar to the fact of the existence of frontal solutions of the type of thermal waves in the theory of nonlinear thermal conductivity [11]. In both cases the generalized solutions of nonlinear parabolic equations (systems) contain a surface of a weak discontinuity, i.e., a front, at which a constant solution is joined with a variable solution. In [12] a connection was established between solutions of the type of thermal waves and the existence of singular solutions of the corresponding differential equations (systems). With a consideration of problems of the theory of the boundary layer of dilatant liquids, this connection is also evident.

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EQUILIBRIUM AND STABILITY OF AN INCOMPRESSIBLE FLUID DROP

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Equilibrium shapes and stability of axially symmetric drops were investigated in detail in [1-3]. The papers [4-6] were devoted to conditions of drop breakup following slow growth. Based on the results of [6], a mechanism was suggested [7] for determining the surface-tension coefficient of the fluid by the height of the drop at the moment of break-up. Below we consider equilibrium and stability of an axially symmetric drop, adjacent to a bulk incompressible fluid and bounded by a planar free surface. Unlike [1-7], in studying the stability of this system [8] it is necessary to take into account perturbations varying the volume of the drop. Therefore the class of stable equilibrium shapes is narrowed down.

1. Let some volume Q of an incompressible fluid be in a uniform field of mass forces and be confined by solid walls of a container S and by free surfaces Σ and Σ_1. The surface Σ_1 is planar, and Σ confines that part of the volume, protruding in the form of a drop at the outer surface of the container walls. We assume that the wall surface near the contour L is the base of the drop and the wetting characteristic of this part of the wall is axially symmetric, while the symmetry axis is parallel to the direction of the gravity field **g**. We assume that the drop is also axially symmetric. The contour radius of the drop base is denoted by R_0, and the drop height by H. We introduce a cylindrical coordinate system $\{r, z, \theta\}$ with origin at the center of the drop base and a Z axis along the symmetry axis inside the volume Q (Fig. 1). The coordinates r and z are dimensionless: $r = R/R_0$, $z = Z/R_0$.

We denote by s the path length measured from the plane of the drop along the meridian. The meridian coordinates are given parametrically

$$r = r(s),\ z = z(s),\ 0 \leqslant s \leqslant l,$$

where l is the total length of the drop meridian.

The functions r(s), z(s) satisfy the well-known [1] system of ordinary differential equations

$$r''(s) = -z'(s)q(s,\ \beta,\ \eta), z''(s) = r'(s)q(s,\ \beta,\ \eta), \qquad (1.1)$$
$$q(s,\ \beta,\ \eta) = \beta z(s) + \eta - z'(s)/r(s)$$

and the boundary conditions ($-h$ is the ordinate of the pole of the drop)

$$r(0) = 0,\ z(0) = -h,\ r(l) = 1,\ z(l) = 0. \qquad (1.2)$$

The dimensionless parameters β, η of the system (1.1) are

$$\beta = \rho g R_0^2/\sigma, \quad \eta = p_0 R_0/\sigma, \qquad (1.3)$$

where ρ is the fluid density, g is the acceleration projection of the gravity force **g** on the Z axis, σ is the surface-tension coefficient of the drop, and p_0 is the pressure at the base plane of the drop ($z \equiv 0$).

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